Rational Functions

A **rational function** is any function where the output can be defined by dividing a polynomial function by a polynomial function.

Simple things to understand about fractions and division as you consider properties of rational functions:

1) There is no such thing as a quotient with a zero in the denominator. i.e. any expression written with division by zero is not a number.

2) When the numerator is considerably larger than the denominator then the quotient will have a large absolute value.

3) When the denominator is considerably larger than the numerator then the quotient will have a small absolute value.

Given the function $f(x) = \frac{x+1}{x^2-4}$

What is the domain of this function? What is the x-intercept for this function?

What happens to the output values of the function as the input values get bigger to the right and to the left?

$f : \{(4, \frac{5}{12})\,, (8, \frac{9}{60}), (10, \frac{11}{96})\,, \ldots\ldots, (100, \frac{101}{9996}), \ldots\ldots\}$

$\boldsymbol{x \to \infty\,,\ f(x) \to 0}$

What happens to the output values of the function as the input values get bigger to the right and to the left?

$f : \{(-4, -\frac{3}{12})\,, (-8, -\frac{7}{60}), \ldots., (-100, -\frac{99}{9996}), \ldots.\}$

$\boldsymbol{x \to -\infty\,,\ f(x) \to 0}$

Because the degree of the bottom is larger than the degree of the top, the absolute value of the denominator becomes significantly larger than the numerator and when you divide by large numbers your quotient is small.

Given the function $f(x) = \frac{x+1}{x^2 - 4}$

What happens to the output values of the function as the input values get closer to 2?

$f : \{(1.9, -7.4), (1.99, -74.9) \ldots (1.9999, -7499.9) \ldots\}$

$\boldsymbol{x \rightarrow 2^{-}\ ,\ f(x) \rightarrow -\infty}$

$f : \{(2.1, 7.56), (2.01, 75.06) \ldots (2.0001, 7500.06) \ldots\}$

$\boldsymbol{x \rightarrow 2^{+}\ ,\ f(x) \rightarrow \infty}$

What happens to the output values of the function as the input values get closer to -2?

$f : \{(-2.1, -2.7), (-2.01, -25.2) \ldots (-2.001, -250.2) \ldots\}$

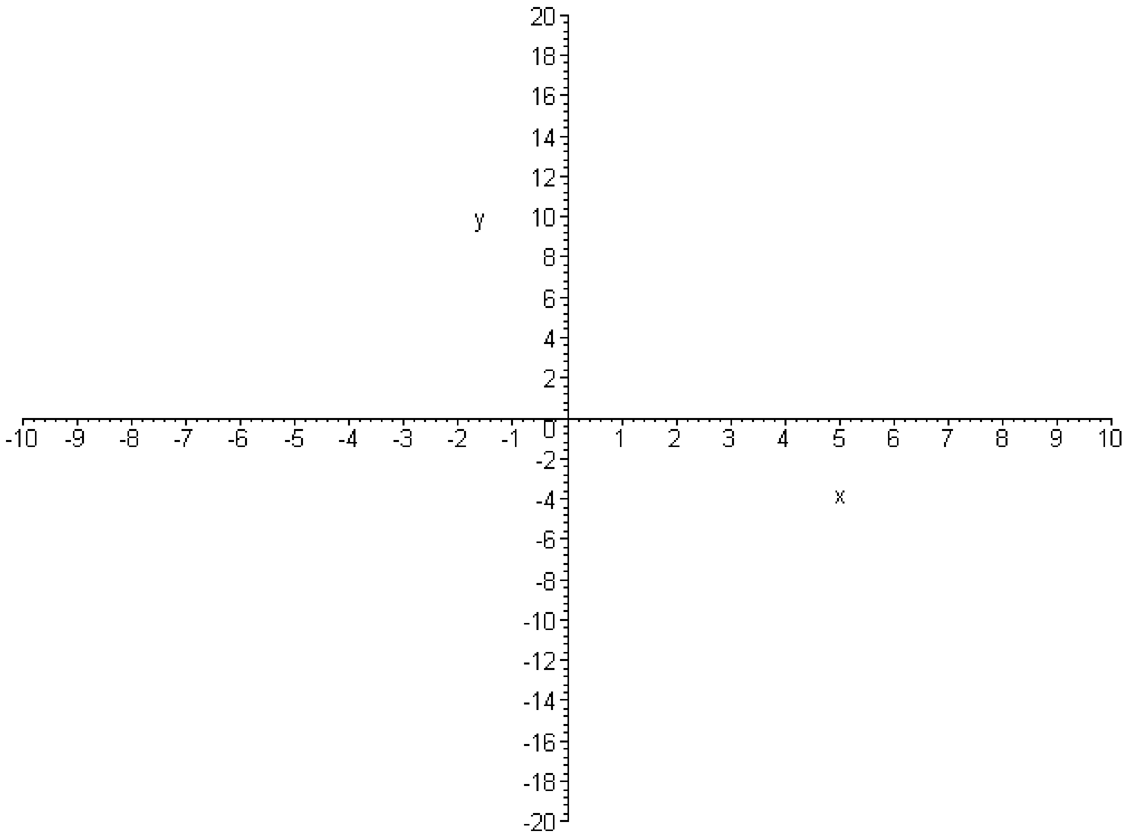
$\boldsymbol{x \rightarrow -2^{-}\ ,\ f(x) \rightarrow -\infty}$

$f : \{(-1.9, 2.3), (-1.99, 24.8) \ldots (-1.999, 249.8) \ldots\}$

$\boldsymbol{x \rightarrow -2^{+}\ ,\ f(x) \rightarrow \infty}$

The function is undefined at 2 and -2 but as your input moves closer and closer to 2 or negative 2 then the denominator moves closer and closer to 0 making the quotient get larger and larger in a positive or negative direction.

$$f(x) = \frac{x+1}{x^2-4}$$



To find the **vertical asymptotes** of a reduced rational function, you find the zeros of the denominator. (A function is undefined at a vertical asymptote.)

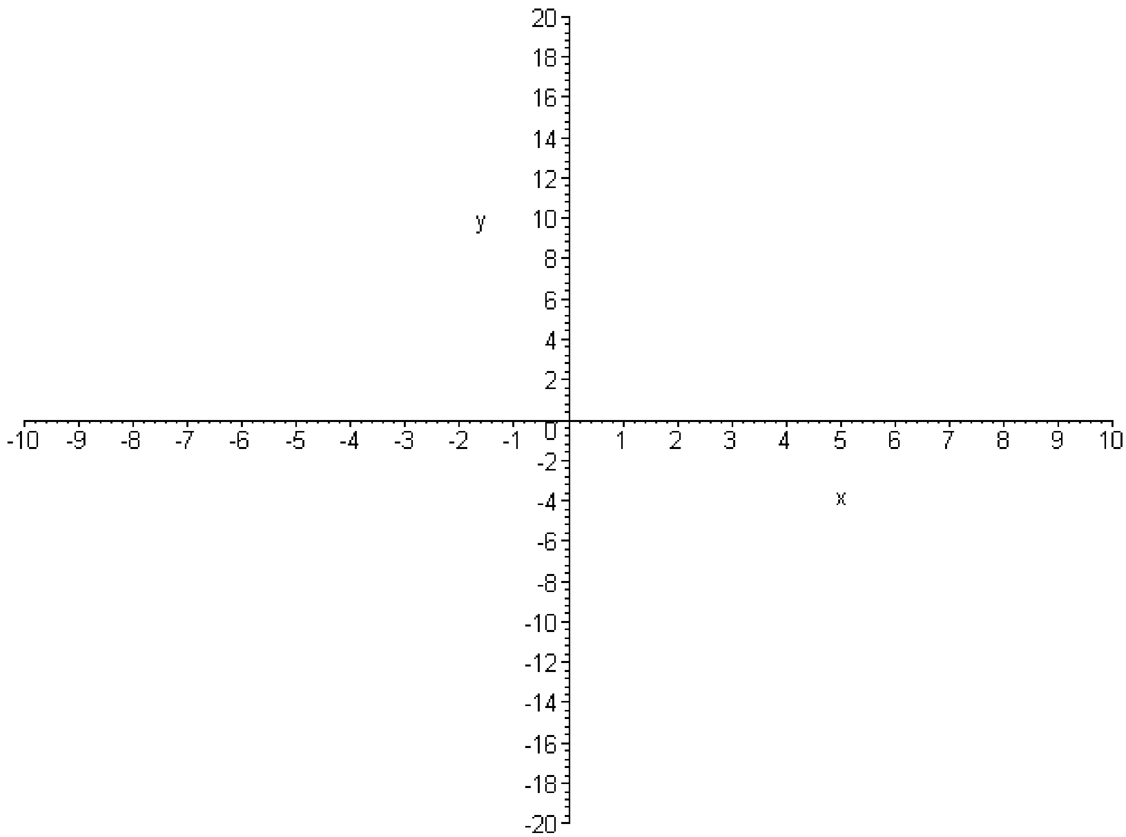
A **horizontal asymptote** will occur anytime the degree of the numerator is less than or equal to the degree of the denominator. If the degree of the numerator is less than the degree of the denominator then the line $y = 0$ (the x-axis) will be a horizontal asymptote. If the degree of the numerator is equal to the degree of the denominator then the line $y = \frac{\text{Leading coefficient top}}{\text{Leading coefficient bottom}}$ will be a horizontal asymptote.

An **oblique asymptote** will occur anytime the degree of the numerator is greater by 1 than the degree of the denominator. To find the equation of an oblique asymptote, you divide the numerator by the denominator and ignore the remainder.

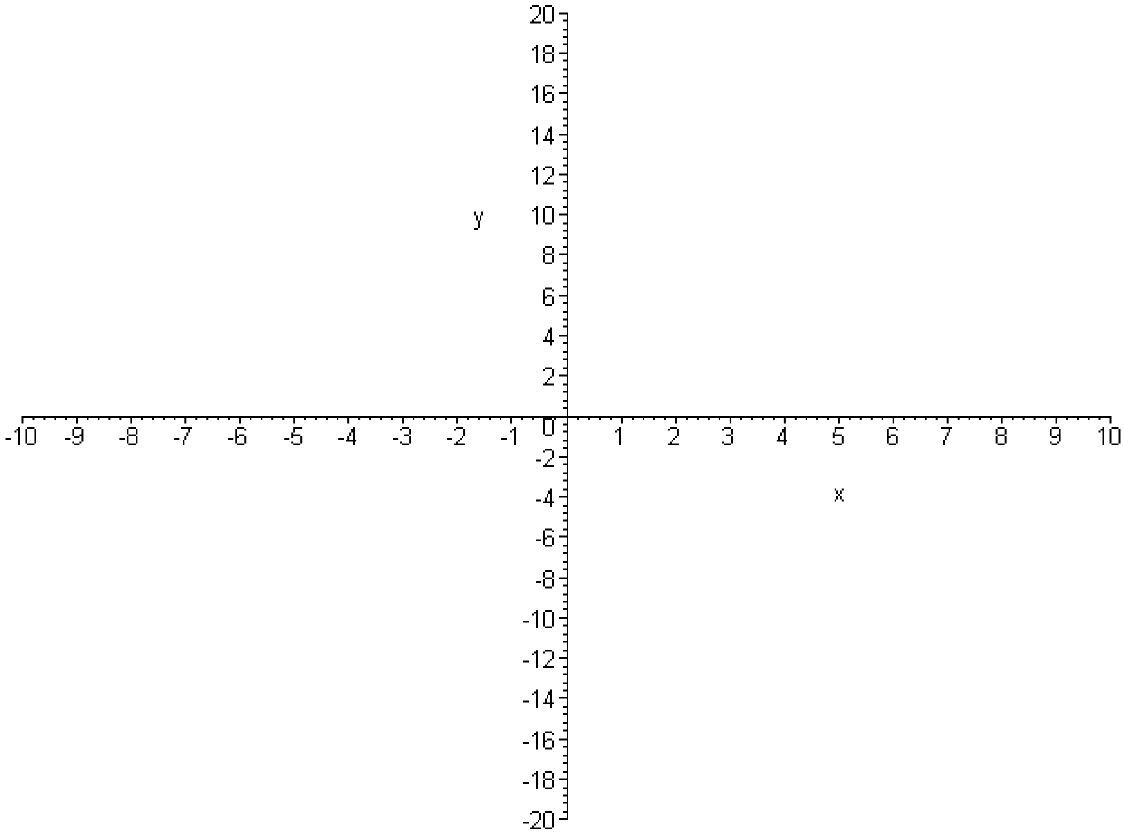
Example: $r(x) = \frac{x^3-2x+1}{x^2-x+2}$

Try graphing the following rational functions:

$$f(x) = \frac{4x^2 - 9}{2x^2 + 5x + 3}$$



$$g(x) = \frac{x-1}{x^3 - x}$$



$$h(x) = \frac{x^2 + 3x + 2}{x - 2}$$